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Permutation and Combination are the most fundamental concepts in mathematics related to picking items from a group or set. Permutation is arranging items considering the order of selection from a certain group.Combination is selecting items without considering order.For example, in the below diagram, PQ and QP are different in permutation but same in combination. Therefore we have more permutations than combinations.Permutations and CombinationsPermutation MeaningPermutation is the distinct interpretations of a provided number of components carried one by one, or some, or all at a time. For example, if we have two components A and B, then there are two likely performances, AB and BA.A numeral of permutations when ‘r’ components are positioned out of a total of ‘n’ components is n Pr. For example, let n = 3 (A, B, and C) and r = 2 (All permutations of size 2). Then there are 3P2 such permutations, which is equal to 6. These six permutations are AB, AC, BA, BC, CA, and CB. The six permutations of A, B, and C taken three at a time are shown in the image added below:Permutation MeaningPermutation FormulaPermutation formula is used to find the number of ways to pick r things out of n different things in a specific order, and replacement is not allowed, and is given as follows:Permutation FormulaExplanation of Permutation FormulaAs we know, permutation is an arrangement of r things out of n where the order of arrangement is important(AB and BA are two different permutations). If there are three different numerals 1, 2 and 3 and if someone is curious to permute the numerals taking 2 at a moment, it shows (1, 2), (1, 3), (2, 1), (2, 3), (3, 1), and (3, 2). That it can be accomplished in 6 methods. Here, (1, 2) and (2, 1) are distinct. Again, if these 3 numerals shall be put handling all at a time, then the interpretations will be (1, 2, 3), (1, 3, 2), (2, 1, 3), (2, 3, 1), (3, 1, 2) and (3, 2, 1) i.e. in 6 ways. In general, n distinct things can be set taking r (r < n) at a time in n(n - 1)(n - 2)...(n - r + 1) ways. In fact, the first thing can be any of the n things. Now, after choosing the first thing, the second thing will be any of the remaining n - 1 things. Likewise, the third thing can be any of the remaining n - 2 things. Alike, the rth thing can be any of the remaining n - (r - 1) things. Hence, the entire number of permutations of n distinct things carrying r at a time is n(n - 1)(n - 2)...[n - (r - 1)], which is written as n Pr. Or, in other words, \bold{{}^nP_r = \frac{n!}{(n-r)!} } Combination MeaningIt is the distinct sections of a shared number of components carried one by one, or some, or all at a time. For example, if there are two components A and B, then there is only one way to select two things, select both of them.For example, let n = 3 (A, B, and C) and r = 2 (All combinations of size 2). Then there are 3C2 such combinations, which is equal to 3. These three combinations are AB, AC, and BC.Here, the combination of any two letters out of three letters A, B, and C is shown below, we notice that in combination the order in which A and B are taken is not important as AB and BA represent the same combination.Combination Example Note: In the same example, we have distinct points for permutation and combination. For, AB and BA are two distinct items i.e., two distinct permutation, but for selecting, AB and BA are the same i.e., same combination.Combination FormulaCombination Formula is used to choose ‘r’ components out of a total number of ‘n’ components, and is given by:Combination FormulaUsing the above formula for r and (n-r), we get the same result. Thus,\bold{{}^nC_r = {}^nC_{(n-r)}} Explanation of Combination FormulaCombination, on the further hand, is a type of pack. Again, out of those three numbers 1, 2, and 3 if sets are created with two numbers, then the combinations are (1, 2), (1, 3), and (2, 3). Here, (1, 2) and (2, 1) are identical, unlike permutations where they are distinct. This is written as 3C2. In general, the number of combinations of n distinct things taken r at a time is, \bold{{}^nC_r = \frac{n!}{r!\times(n-r)!} = \frac{{}^nP_r}{r!}}Derivation of Permutation and Combination FormulasWe can derive these Permutation and Combination formulas using the basic counting methods, as these formulas represent the same thing. Derivation of these formulas is as follows:Derivation of Permutations FormulaPermutation is selecting r distinct objects from n objects without replacement, and where the order of selection is important. By the fundamental theorem of counting and the definition of permutation, we getP (n, r) = n . (n-1) . (n-2) . (n-3). . . . .(n-(r+1))By multiplying and dividing above with (n-r)! = (n-r).(n-r-1).(n-r-2). . . . .3. 2. 1, we getP (n, r) = [n.(n−1).(n−2)....(nr+1)[(n−r)(n−r−1)(n-r)!] / (n-r)!⇒ P (n, r) = n!/(n−r)!Thus, the formula for P (n, r) is derived.Derivation of Combinations FormulaCombination is choosing r items out of n items when the order of selection is of no importance. Its formula is calculated as,C(n, r) = Total Number of Permutations /Number of ways to arrange r different objects. [Since by the fundamental theorem of counting, we know that number of ways to arrange r different objects in r ways = r!]C(n,r) = P (n, r)/ r!⇒ C(n,r) = n!/(n−r)!r!Thus, the formula for Combination i.e., C(n, r) is derived.Read in Detail - [Combinations Formulas in Maths]Permutation Vs CombinationDifferences between permutation and combination can be understood by the following table:PermutationCombinationIn Permutation order of arrangement is important.For example, AB and BA are different combinations.In Combination order of arrangement is not important.For example, AB and BA are the same combinations.A permutation is used when different kinds of things are to be sorted or arranged.Combinations are used when the same kind of things are tobe sorted.Permutation of two things out of three given things a, b, c is ab, ba, bc, cb, ac, ca.The combination of two things from the three given thingsa, b, c is ab, bc, ca.Formula for permuation is: n Pr = n!/(n - r)!The formula for Combination is: n Cr = n! /{r! × (n - r)!}Also Check,Solved Examples on Permutation and CombinationExample 1: Find the number of permutations and combinations of n = 9 and r = 3.Solution: Given, n = 9, r = 3Using the formula given above:For Permutation:nPr = (n!) / (n - r)! ⇒ nPr = (9!) / (9 - 3)! ⇒ nPr = 9! / 6! = (9 × 8 × 7 × 6!)/ 6! ⇒ nPr = 504For Combination:nCr = n!/r!(n − r)!⇒ nCr = 9!/3!(9 − 3)!⇒ nCr = 9!/3!(6)!⇒ nCr = 9 × 8 × 7 × 6!/3!(6)!⇒ nCr = 84Example 2: In how many ways can a committee consisting of 4 men and 2 women can be chosen from 6 men and 5 women?Solution:Choose 4 men out of 6 men = 6C4 ways = 15 waysChoose 2 women out of 5 women = 5C2 ways = 10 waysThe committee can be chosen in 6C4 × 5C2 = 150 ways.Example 3: In how many ways can 5 different books be arranged on a shelf?Solution: This is a permutation problem because the order of the books matters. Using the permutation formula, we get:5P5 = 5! / (5 - 5)! = 5! / 0! = 5 x 4 x 3 x 2 x 1 = 120Therefore, there are 120 ways to arrange 5 different books on a shelf.Example 4: How many 3-letter words can be formed using the letters from the word "FABLE"?Solution: This is a permutation problem because the order of the letters matters. Using the permutation formula, we get:5P3 = 5! / (5 - 3)! = 5! / 2! = 5 x 4 x 3 = 60Therefore, there are 60 3-letter words that can be formed using the letters from the word "FABLE".Example 5: A committee of 5 members is to be formed from a group of 10 people. In how many ways can this be done?Solution: This is a combination problem because the order of the members doesn't matter. Using the combination formula, we get:10C5 = 10! / (5! x (10 - 5)!) = 10! / (5! x 5!) ⇒10C5= (10 x 9 x 8 x 7 x 6) / (5 x 4 x 3 x 2 x 1) = 252Therefore, there are 252 ways to form a committee of 5 members from a group of 10 people.Example 6: A pizza restaurant offers 4 different toppings for their pizzas. If a customer wants to order a pizza with exactly 2 toppings, in how many ways can this be done?Solution: This is a combination problem because the order of the toppings doesn't matter. Using the combination formula, we get:4C2 = 4! / (2! x (4 - 2)!) = 4! / (2! x 2!) = (4 x 3) / (2 x 1) = 6Therefore, there are 6 ways to order a pizza with exactly 2 toppings from 4 different toppings.Example 7: How many considerable words can be created by using 2 letters from the term“LOVE”?Solution: The term “LOVE” has 4 distinct letters.Therefore, required number of words = 4P2 = 4! / (4 – 2)!Required number of words = 4! / 2! = 24 / 2⇒ Required number of words = 12Example 8: Out of 5 consonants and 3 vowels, how many words of 3 consonants and 2 vowels can be formed?Solution:Number of ways of choosing 3 consonants from 5 = 5C3Number of ways of choosing 2 vowels from 3 = 3C2Number of ways of choosing 3 consonants from 2 and 2 vowels from 3 = 5C3 × 3C2⇒ Required number = 10 × 3= 30It means we can have 30 groups where each group contains a total of 5 letters (3 consonants and 2 vowels).Number of ways of arranging 5 letters among themselves= 5! = 5 × 4 × 3 × 2 × 1 = 120Hence, the required number of ways = 30 × 120⇒ Required number of ways = 3600Example 9: How many different combinations do you get if you have 5 items and choose 4?Solution:Insert the given numbers into the combinations equation and solve. “n” is the number of items that are in the set (5 in this example); “r” is the number of items you’re choosing (4 in this example):C(n, r) = n! / r! (n – r)! ⇒ nCr = 5! / 4! (5 – 4)!⇒ nCr = (5 × 4 × 3 × 2 × 1) / (4 × 3 × 2 × 1 × 1)⇒ nCr = 120/24 ⇒ nCr = 5The solution is 5.Example 10: Out of 6 consonants and 3 vowels, how many expressions of 2 consonants and 1 vowel can be created?Solution:Number of ways of selecting 2 consonants from 6 = 6C2Number of ways of selecting 1 vowels from 3 = 3C1Number of ways of selecting 3 consonants from 7 and 2 vowels from 4.⇒ Required ways = 6C2 × 3C1⇒ Required ways = 15 × 3⇒ Required ways= 45It means we can have 45 groups where each group contains a total of 3 letters (2 consonants and 1 vowels).Number of ways of arranging 3 letters among themselves = 3! = 3 × 2 × 1⇒ Required ways to arrenge three letters = 6Hence, the required number of ways = 45 × 6⇒ Required ways = 270Example 11: In how many distinct forms can the letters of the term 'PHONE' be organized so that the vowels consistently come together?Solution:The word 'PHONE' has 5 letters. It has the vowels 'O','E', in it and these 2 vowels should consistently come jointly. Thus these two vowels can be grouped and viewed as a single letter. That is, PHN(OE).Therefore we can take total letters like 4 and all these letters are distinct.Number of methods to organize these letters = 4! = 4 × 3 × 2 × 1⇒ Required ways arrenge letters = 24All the 2 vowels (OE) are distinct.Number of ways to arrange these vowels among themselves = 2! = 2 × 1⇒ Required ways to arrange vowels = 2Hence, the required number of ways = 24 × 2⇒ Required ways = 48.Practice Questions - Permutations and CombinationsQuestion 1: How many ways can 5 books be arranged on a shelf?Question 2: In how many ways can a committee of 3 be chosen from a group of 10 people?Question 3: How many 4-digit PIN codes can be created using the digits 0-9 if repetition is allowed?Question 4: A pizza shop offers 8 toppings. How many different 3-topping pizzas can be made?Question 5: In how many ways can 6 people be seated at a round table?Question 6: How many ways are there to select 2 boys and 3 girls from a class of 5 boys and 6 girls?Question 7: How many different 5-card hands can be dealt from a standard 52-card deck?Question 8: In how many ways can the letters of the word "MATHEMATICS" be arranged?Question 9: A bag contains 5 red marbles, 3 blue marbles, and 2 green marbles. How many ways are there to select 4 marbles?Question 10: How many ways are there to arrange the letters in the word "MISSISSIPPI"?Permutations and combinations are fundamental concepts in probability and statistics used to calculate the number of possible outcomes in various scenarios. Permutations deal with arrangements where order matters, calculated using the formula P(n,r) = n! / (n-r)!, where n is the total number of items and r is the number being arranged. Combinations, on the other hand, focus on selections where order is irrelevant, using the formula C(n,r) = n! / (r! * (n-r)!). The key difference lies in whether the order of selection is important. Factorials, denoted as n!, play a crucial role in both calculations, representing the product of all positive integers up to and including n. These concepts find applications in diverse fields, from creating PIN codes and arranging seating plans (permutations) to selecting team members and calculating lottery odds (combinations). Understanding when to apply each concept is essential for solving problems involving counting and probability in both academic and real-world contexts. An arrangement of objects in which the order is not important is called a combination. This is different from permutation where the order matters. For example, suppose we are arranging the letters A, B and C. In a permutation, the arrangement ABC and ACB are different. But, in a combination, the arrangements ABC and ACB are the same because the order is not important. What Is The Combination Formula? The number of combinations of n things taken r at a time is written as C(n, r). The following diagram shows the formula for combination. Scroll down the page for more examples and solutions on how to use the combination formula. The number of combinations of selecting k items from a set of n distinct items is denoted in various ways: \(C(n,k)\) or \(_nC_k\) or \((^n_k)\) (read as “n choose k”) The formula for Combinations is: \((^n_k)=\frac{n!}{k!(n-k)!}\) Where: n = the total number of items in the set. k = the number of items you want to choose. ! = factorial If you are not familiar with the n! (n factorial notation) then have a look the factorial lesson How To Use The Combination Formula To Solve Word Problems? Example: In how many ways can a coach choose three swimmers from among five swimmers? Solution: There are 5 swimmers to be taken 3 at a time. Using the formula: The coach can choose the swimmers in 10 ways. Example: Six friends want to play enough games of chess to be sure every one plays everyone else. How many games will they have to play? Solution: There are 6 players to be taken 2 at a time. Using the formula: They will need to play 15 games. Example: In a lottery, each ticket has 5 one-digit numbers 0-9 on it. a) You win if your ticket has the digits in any order. What are your changes of winning? b) You would win only if your ticket has the digits in the required order. What are your chances of winning? Solution: There are 10 digits to be taken 5 at a time. a) Using the formula: The chances of winning are 1 out of 252. b) Since the order matters, we should use permutation instead of combination. P(10, 5) = 10 x 9 x 8 x 7 x 6 = 30240 The chances of winning are 1 out of 30240. How To Evaluate Combinations As Well As Solve Counting Problems Using Combinations? A combination is a grouping or subset of items. For a combination, the order does not matter. How many committees of 3 can be formed from a group of 4 students? This is a combination and can be written as C(4,3) or 4C3 or \(\left({\begin{array}{*{20}{c}}4\\3\end{array}} \right)\). Examples: The soccer team has 20 players. There are always 11 players on the field. How many different groups of players can be on the field at any one time? A student need 8 more classes to complete her degree. If she met the prerequisites for all the courses, how many ways can she take 4 classes next semester? There are 4 men and 5 women in a small office. The customer wants a site visit from a group of 2 man and 2 women. How many different groups can be formed from the office? Show Video Lesson Examples: A museum has 7 paintings by Picasso and wants to arrange 3 of them on the same wall. How many ways are there to do this? How many ways can you arrange the letters in the word LOLLIPOP? A person playing poker is dealt 5 cards. How many different hands could the player have been dealt? Show Video Lesson How To Solve Combination Problems That Involve Selecting Groups Based On Conditional Criteria? Example: A bucket contains the following marbles: 4 red, 3 blue, 4 green, and 3 yellow making 14 total marbles. Each marble is labeled with a number so they can be distinguished. How many sets/groups of 4 marbles are possible? How many sets/groups of 4 are there such that each one is a different color? How many sets of 4 are there in which at least 2 are red? How many sets of 4 are there in which none are red, but at least one is green? Show Video Lesson Try out our new and fun Fraction Concoction Game. Add and subtract fractions to make exciting fraction concoctions following a recipe. There are four levels of difficulty: Easy, medium, hard and insane. Practice the basics of fraction addition and subtraction or challenge yourself with the insane level. We welcome your feedback, comments and questions about this site or page. Please submit your feedback or enquiries via our Feedback page. The factorial function (symbol: !) says to multiply all whole numbers from our chosen number down to 1. Examples: 4! = 4 × 3 × 2 × 1 = 24 7! = 7 × 6 × 5 × 4 × 3 × 2 × 1 = 5040 1! = 1 We usually say (for example) 4! as "4 factorial", but some people say "4 shriek" or "4 bang". Each factorial builds on the previous one, making calculations easier: As a table: n n! 1 1 1 1 2 2 × 1 = 2 × 1! = 2 3 3 × 2 × 1 = 3 × 2! = 6 4 4 × 3 × 2 × 1 = 4 × 3! = 24 5 5 × 4 × 3 × 2 × 1 = 5 × 4! = 120 6 etc etc To work out 6!, multiply 120 by 6 to get 720 To work out 7!, multiply 720 by 7 to get 5040 And so on 10! = 10 × 9! 10! = 10 × 362,880 = 3,628,800 So the rule is: n! = n × (n−1)! Which says "the factorial of any number is that number times the factorial of (that number minus 1)" So 10! = 10 × 9!, ... and 125! = 125 × 124!, etc. What About "0!" Zero Factorial is interesting ... it is generally agreed that 0! = 1. It may seem funny that multiplying no numbers together results in 1, but let's follow the pattern backwards from, say, 4! like this: And in many equations using 0! = 1 just makes sense. Example: how many ways can we arrange letters (without repeating)? For 1 letter "a" there is only 1 way: a For 2 letters "ab" there are 1×2=2 ways: ab, ba For 3 letters "abc" there are 1×2×3=6 ways: abc acb cab bac bca cba For 4 letters "abcd" there are 1×2×3×4=24 ways: (try it yourself!) etc The formula is simply n! Now ... how many ways can we arrange no letters? Just one way, an empty space: So 0! = 1 Where is Factorial Used? One area they are used is in Combinations and Permutations. We had an example above, and here is a slightly different example: Example: How many different ways can 7 people come 1st, 2nd and 3rd? The list is quite long, if the 7 people are called a,b,c,d,e,f and g then the list includes: abc, abd, abe, abf, abg, acb, acd, ace, acf, ... etc. The formula is 7!(7−3)! = 7!4! Let us write the multiplies out in full: 7 × 6 × 5 × 4 × 3 × 2 × 14 × 3 × 2 × 1 = 7 × 6 × 5 That was neat. The 4 × 3 × 2 × 1 "cancelled out", leaving only 7 × 6 × 5. And: 7 × 6 × 5 = 210 So there are 210 different ways that 7 people could come 1st, 2nd and 3rd. Done! Using our knowledge from the previous example we can jump straight to this: 100!98! = 100 × 99 = 9900 Factorials also appear in probability and statistics, such as calculating lottery odds. A Small List n n! 0 1 1 1 2 2 3 6 4 24 5 120 6 720 7 5,040 8 40,320 9 362,880 10 3,628,800 11 39,916,800 12 479,001,600 13 6,227,020,800 14 87,178,291,200 15 1,307,674,368,000 16 20,922,789,888,000 17 355,687,428,096,000 18 6,402,373,705,728,000 19 121,645,100,408,832,000 20 2,432,902,008,176,640,000 21 51,090,942,171,709,440,000 22 1,124,000,727,777,607,680,000 23 25,852,016,738,884,976,640,000 24 620,448,401,733,239,439,360,000 25 15,511,210,043,330,985,984,000,000 As you can see, it gets big quickly. If you need more, try the Full Precision Calculator. Interesting Facts Six weeks is exactly 10! seconds (=3,628,800) Here is why: Seconds in 6 weeks: 60 × 60 × 24 × 7 × 6 Factor some numbers: (2 × 3 × 10) × (3 × 4 × 5) × (8 × 3) × 7 × 6 Rearrange: 2 × 3 × 4 × 5 × 6 × 7 × 8 × 3 × 3 × 10 Last 3×3=9: 2 × 3 × 4 × 5 × 6 × 7 × 8 × 9 × 10 There are 52! ways to shuffle a deck of cards. That is 8.0658175... × 1067 Just shuffle a deckof cards and it is likely that you are the first person ever with that particular order. There are about 60! atoms in the observable Universe. 60! is about 8.320987... × 1081 and the current estimates are between 1078 to 1082 atoms in the observable Universe. 70! is approximately 1.197857... x 10100, which is just larger than a Googol (the digit 1 followed by one hundred zeros). 100! is approximately 9.332621544394415268169923856 x 10157 200! is approximately 7.886578673647905035523632139 x 10374 n! ≈ (ne)n √2πn The "≈" means "approximately equal to". Let us see how good it is: n n! Close Formula (to 2 Decimals) Accuracy (to 4 Decimals) 1 1 0.92 0.9221 2 2 1.92 0.9595 3 6 5.84 0.9727 4 24 23.51 0.9794 5 120 118.02 0.9835 6 720 710.08 0.9862 7 5040 4980.40 0.9882 8 40320 39902.40 0.9896 9 362880 359536.87 0.9908 10 3628800 3598695.62 0.9917 11 39916800 39615625.05 0.9925 12 479001600 475687486.47 0.9931 If you don't need perfect accuracy this may be useful. Note: it is called "Stirling's approximation" and is based on a simplified version of the Gamma Function. What About Negatives? Can we have factorials for negative numbers? Yes ... but not for negative integers. Negative integer factorials, like (−1)!, (−2)!, etc, are undefined. Let's start with 3! = 3 × 2 × 1 = 6 and go down: 2! = 3! / 3 = 6 / 3 = 2 1! = 2! / 2 = 2 / 2 = 1 0! = 1! / 1 = 1 / 1 = 1 which is why 0!=1 (−1)! = 0! / 0 = 1 / 0 = ? oops, dividing by zero is undefined And from there on down all integer factorials are undefined. What About Decimals? Can we have factorials for numbers like 0.5 or −3.217? Yes we can! But we need to use the Gamma Function (advanced topic). Factorials can also be negative (except for negative integers). But I can tell you the factorial of half (½) is half of the square root of pi . Here are some "half-integer" factorials: (−½)! = √π (½)! = (½)√π (3/2)! = (3/4)√π (5/2)! = (15/8)√π It still follows the rule that "the factorial of any number is that number times the factorial of (1 smaller than that number)", because (3/2)! = (3/2) × (1/2)! (5/2)! = (5/2) × (3/2)! Can you figure out what (7/2)! is? A double factorial is like a normal factorial but we skip every second number: 8!! = 8 × 6 × 4 × 2 = 384 9!! = 9 × 7 × 5 × 3 × 1 = 945 Notice how we multiply all even, or all odd, numbers. Note: if we want to apply factorial twice we write (n!)! 2229, 2230, 7006, 2231, 7007, 9080, 9081, 9082, 9083, 9084 Copyright © 2025 Rod Pierce Combinations are also called selections. Combinations correspond to the selection of things from a given set of things. Here we do not intend to arrange things. We intend to select them. We denote the number of unique r-selections or combinations out of a group of n objects by \(^n{C_r}\). Combinations are different from arrangements or permutations. Let us learn more about how to calculate combinations, combinations formula, differences between permutation and combinations, with the help of examples, FAQs. What Are Combinations? Combinations are selections made by taking some or all of a number of objects, irrespective of their arrangements. The number of combinations of n different things taken r at a time, denoted by \(^n{C_r}\) and it is given by, \(^n{C_r} = \dfrac{n!}{r!(n-r)!}\),where 0 ≤ r ≤ n. This forms the general combination formula which is \(^n C_r\) formula. This formula to find the number of combinations by using r objects from the n objects, is also referred as the ncr formula. What Is Combinations Formula? The combinations formula is used to easily find the number of possible different groups of r objects each, which can be formed from the available n different objects. Combinations formula is the factorial of n, divided by the product of the factorial of r, and the factorial of the difference of n and r respectively. Combinations Formula: \(^nC_r = \dfrac{n!}{r!.(n - r)!}\) The combinations formula is also referred to as ncr formula. To use the combinations formula we need to know the meaning of factorial, and we have n! = 1 × 2 × 3 × .... (n - 1) × n. Combinations as Selections Suppose we have a set of 6 letters { A,B,C,D,E,F}. In how many ways can we select a group of 3 letters from this set? Suppose we find the number of arrangements of 3 letters possible from those 6 letters. That number would be 6P\(_3\). Consider the permutations that contain the letters A, B, and C. These are 3! = 6 ways, namely ABC, ACB, BAC, BCA, CAB, and CBA. Now, what we want is the number of combinations and not the number of arrangements. In other words, the 6 permutations listed above would correspond to a single combination. Differently put, the order of things is not important; only the group/combination matters now in our selection. This means that the total number of combinations of 3 letters from the set of 6 letters available to us would be 6P\(_3\)/3! ways, since each combination is counted 3! times in the list of permutations. Thus, if we denote the number of combinations of 6 things taken 3 at a time by 6C\(_3\), we have: \(^6{C_3}= \dfrac{^6{P_3}}{3}\). This is also said as 6 choose 3. A few important results on combinations are as follows: The number of ways of selecting n objects out of n objects is:\(^n{C_n} = \frac{{n!}}{{n!\left({n - n} \right)!}} = \frac{{n!}}{{n!0!}} = 1\) The number of ways of selecting 0 objects out of n objects is:\(^n{C_0} = \frac{{n!}}{{0!\left({n - 0} \right)!}} = \frac{{n!}}{{0!n!}} = 1\) The number of ways of selecting 1 object out of n objects is: \(^n{C_1} = \frac{{n!}}{{1!\left({n - 1} \right)!}} = \frac{{n \times \left({n - 1} \right)!}}{{\left({n - 1} \right)!}} = n\) \(^n{C_r} = ^n{C_{n-r}}\) \(^n{C_r} +^n{C_{r-1}} = ^{n+1}{C_{r}}\) How To Apply Combinations Formula? We calculate combinations using the combinations formula, and by using factorials and in terms of permutations. In general, suppose we have n things available to us, and we want to find the number of ways in which we can select r things out of these n things. We first find the number of all the permutations of these n things taken r at a time. That number would be \(^nP_r\) . Now, in this list of \(^nP_r\) permutations, each combination will be counted r! times since r things can be permuted amongst themselves in r! ways. Thus, the total number of permutations and combinations of these n things, taken r at a time, denoted by \(^nC_r\), will be: \(^n{C_r} = \dfrac{^n{P_r}}{r} = \dfrac{n!}{r!(n - r)!}\) Relationship Between Permutations and Combinations Permutation and combination formulas and concepts have a lot of similarities. Suppose that you have n different objects. You have to determine the number of unique r-selections (selections that contain r objects) which can be made from this group of n objects. Think of a group of n people – you have to find the number of unique sub-groups of size r, which can be created from this group. The number of permutations of size r will be \(^n{P_r}\). In the list of \(^n{P_r}\) permutations, each unique selection will be counted r! times, because the objects in an r-selection can be permuted amongst themselves in \(r!\) ways. Thus, the number of unique combinations can be \(\frac{{^n{P_r}}}{{r!}}\). \(^n{C_r} = \dfrac{^nP_r}{r!} = \dfrac{n!}{(n - r)! r!} = \dfrac{n!}{r!(n - r)}\) Examples on Combinations Example 1: Consider the word EDUCATION. This has 9 distinct letters. How many 3-letter permutations (words) can be formed using the letters of this word? We now know how to answer questions like this; the answer in this particular case will be \(^9{P_3}\) Consider the following 3-letter permutations formed using the letters A, E, T from EDUCATION: AET , ATE, EAT, ETA, TAE, TEA These 6 different arrangements correspond to the same selection of letters, which is {A, E, T}. Thus, in the list of all 3-letter permutations, we will find that each unique Combination corresponds to 6 different arrangements. To find the number of unique 3-letter selections, we divide the number of 3-letter permutations by 6. Hence, the number of 3-letter selections will be \(\dfrac{^9P_3}{6}\) = 60480/ 6 = 10,080 Example 2: Out of a group of 5 people, a pair needs to be formed. The number of possible combinations can be calculated as follows. \(^5{C_2} = \dfrac{{5!}}{{2!\left({5 - 2} \right)!}} = \dfrac{{5!}}{{2!3!}} = \dfrac{120}{2 \times 6} = 10\) Example 3:The number of 4-letter Combinations which can be made from the letters of the word DRIVEN is \(^6{C_4} = \dfrac{6!}{4!(6 - 4)!} = \dfrac{6!}{4!2!} = \dfrac{720}{24 \times 2} = 15\) Important Notes Whenever you read the phrase “number of combinations”, think of the phrase “number of selections”. When you are selecting objects, the order of the objects does not matter. For example, XYZ and XZY are different arrangements but have the same selection. The number of combinations of n distinct objects, taken r at a time (where r is less than n), is \(^nC_r = \dfrac{^nP_r}{r!} = \(\dfrac{n!}{r! (n-r)!}\) This result above is derived from the fact that in the list of all permutations of size r, each unique selection is counted r! times. Out of n objects, the number of ways of combinations 0 or n objects is 1; and the number of ways of selecting 1 object or (n - 1) object is n. Out of n objects, the number of ways of selecting 2 objects is \(^n{C_2} = \frac{{n\left({n - 1} \right)}}{2}\). ☛ Also Check: Example 1. In a party of 10 people, each person shakes hands with every other person. How many possible combinations of handshakes can be made? Solution: Each unique handshake corresponds to a unique pair of persons. Also, note that the order of the two people in the pair does not matter. For example, if X and Y are two people, then XY and YX won’t be counted separately; only the pair {X, Y} will be counted. Here we consider the combination of any two people who shake their hand. Thus, we need to find the number of ways in which 2 people can be selected out of 10. Clearly, the answer is: \(^{10}{C_2} = \dfrac{10!}{2!(10 - 2)!} = \dfrac{10!}{2!8!} = 45\) Answer: 45 handshakes in total Example 2. How many diagonals are there in a polygon with n sides?. (Use the concept of combinations to solve this.) Solution: If we select any two vertices of the polygon and join them, we will get either a diagonal or a side of the polygon. Here we can use the concept of combinations. The number of ways of selecting two vertices out of n is \(^n{C_2} = \frac{n(n - 1)}{2}\). Out of these selections, n correspond to the sides of the polygon. Thus, the number of diagonals is: \(\dfrac{n(n - 1)}{2} - n = \dfrac{n(n - 3)}{2}\). The number of diagonals are \(\dfrac{n (n - 3)}{2}\) Example 3. A class has 25 students. For a school event, 10 students need to be chosen from this class. 4 of the students of the class decide that either four of them will participate in the event, or none of them will participate. What are the possible combination of 10 students? Solution: Given the specified constraint, we divide the set of all possible selections of 10 students into two groups: The selections include the 4 students; we already have 4 students – we need to select 6 more students out of the remaining 21 students. This can be done in \(^{21}{C_6}\) ways. Thus, the number of possible selections that include the 4 students is \(^{21}{C_6}\). The selections do not include the 4 students; now we need to select 10 students out of the remaining 21. This can be done in \(^{21}{C_{10}}\) ways. Thus, the number of possible selections that do not include the 4 students is \(^{21}{C_{10}}\). The total number of possible combinations under the specified constraint is \(^{21}{C_6}{ + ^{21}}{C_{10}}\). View Solution > go to slidego to slidego to slide Great learning in high school using simple cues Indulging in rote learning, you are likely to forget concepts. With Cuemath, you will learn visually and be surprised by the outcomes. Book a Free Trial Class FAQs on Combinations Combinations are selections. Selecting r objects out of the given n objects is given by using the factorials. It is denoted by \(^nC_r = \dfrac{n!}{r!.(n - r)!}\). The combinations are the different subgroups that can be formed from the given larger group of objects. How To Use The Combinations Formula? Combinations are calculated using the combination formula \(^nC_r = \dfrac{n!}{r! (n-r)!}\), while we need to choose r items out of n items. Here we use the factorial formula of n! = 1 × 2 × 3 ×.....(n - 1) × n. Does order matter in combinations? No, the order does not matter in combination. Just the number of selections or subgroups matters. The number of dresses in the wardrobe can be selected at random in order. Picking 2 clothes out of 8 from the wardrobe requires \(^8C_2\) ways = \(\dfrac{8!}{2! 6!}\) = 28 ways. Here we consider the set of two and do not look into the order of the selection. What are the possible combinations of 6 numbers? The possible combinations (selections) out of 6 different numbers are as follows: Combination of 1 out of 6 is \(^{6}{C_1}\) Combination of 2 out of 6 is \(^{6}{C_2}\) Combination of 3 out of 6 is \(^{6}{C_3}\) Combination of 4 out of 6 is \(^{6}{C_4}\) Combination of 5 out of 6 is \(^{6}{C_5}\) Combination of 6 out of 6 is \(^{6}{C_6}\) What are the possible combinations out of the digits 1234? The total number of combinations are possible this way: choosing 1 digit out of 4, choosing 2 digits out of 4, choosing 3 digits out of four and choosing 4 digits out of 4 = \(^{4}{C_1}+ ^{4}{C_2} + ^{4}{C_3} + ^{4}{C_4}\) = 4 + 6 + 4 + 4 = 18 ways How Are Permutations and Combinations Related? The permutations and combinations are related using the combination formula \(^n{C_r} = \dfrac{^nP_r}{r!} = \dfrac{n!}{(n - r) r!} = \dfrac{n!}{r!(n - r)}\). The combinations are the selection of r things taken from n different things, and permutation is the different arrangement of those r things. What Are The Differences Between Permutations and Combinations? Permutations are seen as arrangements of r things out of n things, whereas combinations are seen as selections of r things out of n things. \(^nP_r =\dfrac{n!}{r!}\) and \(^nC_r = \dfrac{n!}{r! (n-r)!}\). For the given r things out of n things, the number of permutations are greater than the number of combinations. What Are the Combination Examples? The combination examples include the groups formed from dissimilar obects.The formation of a committee, the sport team, set of different stationary objects, team of people are some of the combination examples. Share — copy and redistribute the material in any medium or format for any purpose, even commercially. Adapt — remix, transform, and build upon the material for any purpose, even commercially. The licensor cannot revoke these freedoms as long as you follow the license terms. Attribution — You must give appropriate credit , provide a link to the license, and indicate if changes were made . You may do so in any reasonable manner, but not in any way that suggests the licensor endorses you or your use. ShareAlike — If you remix, transform, or build upon the material, you must distribute your contributions under the same license as the original. No additional restrictions — You may not apply legal terms or technological measures that legally restrict others from doing anything the license permits. You do not have to comply with the license for elements of the material in the public domain or where your use is permitted by an applicable exception or limitation . No warranties are given. The license may not give you all of the permissions necessary for your intended use. For example, other rights such as publicity, privacy, or moral rights may limit how you use the material. Please read Introduction to Sets first! This activity investigates how many subsets a set has. What is a Subset? A subset is a set contained in another set It is like you can choose ice cream from the following flavors: {banana, chocolate, vanilla} You could choose any one flavor {banana}, {chocolate}, or {vanilla}, Or any two flavors: {banana, chocolate}, {banana, vanilla}, or {chocolate, vanilla}, Or all three flavors (no that isn't greedy), Or you could say "none at all thanks", which is the "empty set": {} Has the subsets: It also has the subsets: {alex, billy} {alex, casey} {billy, dale} etc ... Also: {alex, billy, casey} {alex, billy, dale} etc ... And also: the whole set: {alex, billy, casey, dale} the empty set: {} Now let's start with the Empty Set and move on up ... The Empty Set How many subsets does the empty set have? You could choose: the whole set: {} the empty set: {} But, hang on a minute, in this case those are the same thing! So the empty set really has just 1 subset (which is itself, the empty set). It is like asking "There is nothing available, so what do you choose?" Answer "nothing". That is your only choice. Done. A Set With One Element The set could be anything, but let's just say it is: {apple} How many subsets does the set {apple} have? the whole set: {apple} the empty set: {} And that's all. You can choose the one element, or nothing. So any set with one element will have 2 subsets. A Set With Two Elements Let's add another element to our example set: {apple, banana} How many subsets does the set {apple, banana} have? It could have {apple}, or {banana}, and don't forget: the whole set: {apple, banana} the empty set: {} So a set with two elements has 4 subsets. A Set With Three Elements How about: {apple, banana, cherry} OK, let's be more systematic now, and list the subsets by how many elements they have: Subsets with one element: {apple}, {banana}, {cherry} Subsets with two elements: {apple, banana}, {apple, cherry}, {banana, cherry} And: the whole set: {apple, banana, cherry} the empty set: {} In fact we could put it in a table: List Number of subsets zero elements {} 1 one element {apple}, {banana}, {cherry} 3 two elements {apple, banana}, {apple, cherry}, {banana, cherry} 3 three elements {apple, banana, cherry} 1 Total: 8 (Note: did you see a pattern in the numbers there?) Sets with Four Elements (Your Turn!) Now try to do the same for this set: {apple, banana, cherry, date} Here is a table for you: List Number of subsets zero elements {} one element two elements three elements four elements Total: (Note: if you did this right, there will be a pattern to the numbers.) And now: {apple, banana, cherry, date, egg} Here is a table for you: List Number of subsets zero elements {} one element two elements three elements four elements five elements Total: (Was there a pattern to the numbers?) Sets with Six Elements What about: {apple, banana, cherry, date, egg, fudge} OK ... we don't need to complete a table, because... ... you should be able to see a pattern by now! The first thing to notice is that the total number of subsets doubles each time: A set with n elements has 2n subsets So you should be able to answer: How many subsets are there for a set of 6 elements? _____ How many subsets are there for a set of 7 elements? _____ Another Pattern Now let's think about subsets and sizes: The empty set has just 1 subset: 1 A set with one element has 1 subset with no elements and 1 subset with one element: 1 1 A set with two elements has 1 subset with no elements, 2 subsets with one element and 1 subset with two elements: 1 2 1 A set with three elements has 1 subset with no elements, 3 subsets with one element, 3 subsets with two elements and 1 subset with three elements: 1 3 3 1 and so on! Do you recognize this pattern of numbers? They are the numbers from Pascal's Triangle! This is very useful, because now you can check if you have the right number of subsets. Note: the rows start at 0, and likewise the columns. {apple, banana, cherry} {apple, banana, date} {apple, banana, egg} {apple, cherry, egg} But that is only 4 subsets, how many should there be? Well, you are choosing 3 out of 5, so go to row 5, position 3 of Pascal's Triangle (remember to start counting at 0) to find you need 10 subsets, so you must think harder! In fact these are the results: {apple,banana,cherry} {apple,banana,date} {apple,banana,egg} {apple,cherry,date} {apple,cherry,egg} {apple,date,egg} {banana,cherry,date} {banana,cherry,egg} {banana,date,egg} {cherry,date,egg} Calculating The Numbers Is there a way of calculating the numbers such as 1, 4, 6, 4 and 1 (instead of looking them up in Pascal's Triangle)? Yes, we can find the number of ways of