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Find parametric equations for the tangent line at the point $(\cos(-\frac{4 \pi}{6}), \sin(-\frac{4 \pi}{6}), -\frac{4 \pi}{6}))$ on the curve $x = \cos(t), y = \sin(t), z=t$ I understand that in order to find the solution, I need to use partial derivatives. However, the method in my textbook works for simpler problems -- I seem to be making a calculation error when I try to apply the method to this problem. Can anyone suggest how to approach this problem? I found a very similar problem and solution here, but the solution by the person who answered is hard for me to follow. Unfortunately, I get stuck at the line where he subtracts $\frac{\pi}{6}$ from π within the trigonometric functions. Here is the "simple" method that I was originally using. Any sincere help would be appreciated. Thank you. A point P = (4,−4, 4) is in a plane with a normal vector n→ = (6, 1, 1). What is the equation and the parametric equation of the plane? You can find the equation of the plane first. You already have everything you need, so you can just insert the coordinates of the point and the values in the normal vector into the general equation of a plane: a (x − x0) + b (y − y0) + c (z − z0) = 0 6 (x − 4) + 1 (−y −(−4)) + 1 (z − 4) = 0 6x − 24 − y + 4 + z − 4 = 0 6x − y + z − 24 = 0 You can find another two points in the plane by using the equation of the plane. If you set two of the coordinates to 0, you will find the third coordinate very quickly. For example, you can set x = y = 0 to find z: 6 · 0 − 0 + z − 24 = 0 z = 24. This means that A = (0, 0, 24) is a point in the plane. You can do the same with x = z = 0 to find y: 6 · 0 − y + 0 − 24 = 0 y = −24. Then, B = (0,−24, 0) is a point in the plane as well. You now have three points in the plane, which can be used to create two vectors! That will give you PA→ = (0 − 4, 0 −(−4) , 24 − 4) = (−4, 4, 20) , PB→ = (0 − 4,−24 −(−4) , 0 − 4) = (−4,−20,−4) . PA→ = (0 − 4, 0 −(−4) , 24 − 4) = (−4, 4, 20) , PB→ = (0 − 4,−24 −(−4) , 0 − 4) = (−4,−20,−4) . We'll use the same point-slope formula to define the equation of the tangent line to the parametric curve that we used to define the tangent line to a cartesian curve, which is???y-y_1=m(x-x_1)??? where ???m??? is the slope and ???(x_1,y_1)??? is the point where the tangent line intersects the curve.To find the slope ???m???, we'll use the formula for the derivative of a parametric curve.???m=\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}???Once we find the derivative of the parametric curve using this formula, we'll plug the given point into the derivative to find the slope at that particular point.Then we'll plug the slope and the given point into the point-slope formula for the equation of a line and simplify to get our tangent line equation. ExampleFind the tangent line(s) to the parametric curve at ???(0,4)???.???x=t^5-4t^3??????y=t^2???To find the derivative of the parametric curve, we'll first need to calculate ???dy/dt??? and ???dx/dt???.???x=t^5-4t^3??????\frac{dx}{dt}=5t^4-12t^2???and???y=t^2??????\frac{dy}{dt}=2t???Plugging these into the derivative formula, we get???\frac{dy}{dx}=\frac{2t}{5t^4-12t^2}???We need to plug the given point into the derivative we just found, but the given point is a cartesian point, and we only have ???t??? in our derivative equation. Therefore, in order to plug the given point into the derivative, we need to convert it from a cartesian point into a parameter value for ???t???. To do this, we'll plug the given point into both of the original parametric equations, and look for matching solutions.???x=t^5-4t^3??????0=t^5-4t^3??????0=t^3\left(t^2-4\right)??????t=0,\ \pm2???and???y=t^2??????4=t^2??????t=\pm2??? Once we find the derivative of the parametric curve using this formula, we'll plug the given point into the derivative to find the slope at that particular point. In order for the parameter value to be valid, it has to be a solution in both equations, which means the parameter value we're interested in are ???t=\pm2???. Since we got two solutions, we're going to have two tangent lines.To solve for the slope of each tangent line, we'll plug ???t=\pm2??? into the derivative equation we found above.For ???t=2????:??? f(2)=\frac{2(2)}{5(2)^4-12(2)^2}??????f(2)=\frac{4}{80-48}??????f(2)=\frac18???For ???t=-2????:???f(-2)=\frac{2(-2)}{5(-2)^4-12(-2)^2}??????f(-2)=\frac{-4}{80-48}??????f(-2)=-\frac18???Finally, plugging the slopes we found and the given point ???(0,4)??? into the point-slope formula for the equation of a line, we get the following two tangent lines.???y-y_1=m(x-x_1)??????y-4=\frac18(x-0)??????y=\frac18x+4???and???y-y_1=m(x-x_1)??????y-4=-\frac18(x-0)??????y=-\frac18x+4???The equations can also be manipulated into this form:???8y=x+32???????-x+8y=32???and???8y=-x+32???????x+8y=32??? Show Mobile Notice Show All Notes Hide All Notes Mobile Notice You appear to be on a device with a "narrow" screen width (i.e. you are probably on a mobile phone). Due to the nature of the mathematics on this site it is best viewed in landscape mode. If your device is not in landscape mode many of the equations will run off the side of your device (you should be able to scroll/swipe to see them) and some of the menu items will be cut off due to the narrow screen width. For problems 1 and 2 compute \(\displaystyle \frac{{dy}}{{dx}}\) and \(\displaystyle \frac{{{d^2}y}}{{d{x^2}}}\) for the given set of parametric equations. \(x = 4{t^3} - {t^2} + 7t\hspace{0.5in}\,\,y = {t^4} - 6\) Solution \(x = {{\bf{e}}^{ - 7t}} + 2\hspace{0.5in}\,\,y = 6{{\bf{e}}^{2t}} + {{\bf{e}}^{ - 3t}} - 4t\) Solution For problems 3 and 4 find the equation of the tangent line(s) to the given set of parametric equations at the given point. \(x = 2\cos \left({3t} \right) - 4\sin \left({3t} \right)\hspace{0.25in}y = 3\tan \left({6t} \right)\) at \(\displaystyle t = \frac{\pi }{2}\) Solution \(x = {t^2} - 2t - 11\hspace{0.25in}y = t{\left({t - 4} \right)^3} - 3{t^2}{\left({t - 4} \right)^2} + 7\) at \(\left({ - 3,7} \right)\) Solution Find the values of t that will have horizontal or vertical tangent lines for the following set of parametric equations. \(x = {t^5} - 7{t^4} - 3{t^3}\hspace{0.25in}y = 2\cos \left({3t} \right) + 4t\) Solution A circle can be a set of all curvy points equidistant from a specific point called the circle's centre. This distance between the circle and its centre is known as radius. Parametric equations of a circle help define the parameters and coordinating points of a circle. The parameters of a circle are independent variables. Furthermore, the circle's parametric equations help us represent the curves on the graph, respectively. Parametric Equations of a CircleThe parametric equations help to describe curves in the two-dimensional coordinate system. Most of the time, these parameters are not functions but are independent variables.Parametric Equations for a Standard CircleLet us consider the circle's centre as point O, and line OP is the radius equal to r. It has its centre at the origin (0, 0). In Cartesian coordinates, the equation of a circle with a point (x, y) on it is represented as follows:x2 + y2 = r2Let us consider the point P (x, y) as the coordinates of any point on the circle. We get a right-angled triangle when a perpendicular line is drawn from the point P to the X-axis. Here, an acute angle is formed opposite to the perpendicular line represented as θ. It is called a parameter.With the help of trigonometric ratios, we get the following values of x and y:y = r sin θ and x = r cos θ.Therefore, the parametric equation of a circle that is centred at the origin (0,0) can be given as P (x, y) = P (r cos θ, r sin θ),(Here 0 ≤ θ ≤ 2π.)In other words, it can be said that for a circle centred at the origin, x2 + y2 = r2 is the equation with y = r sin θ and x = r cos θ as its solution. Here, θ is the parameter.For example, if a circle centred at the origin has its radius of 10 cm, the solution of its parametric equation will be x = 10 cos θ and y= 10 sin θ.Parametric Equations for a General CircleFor a general circle, that is, a circle not centred at the origin. We just need to add or subtract a certain fixed value to the coordinates x and y.Let us consider (h, k) as the coordinates of the centre of the circle. Here, we just need to add them to the coordinates x, and y coordinates in the equation, same as that for the circle centred at the origin. Hence, to get the parametric equation, we need to add the coordinates h and k to x = r cos θ, y = r sin θ.Therefore, the parametric equation of the circle that is not centred at the origin of the circle but the coordinates (h, k) can be written as follows:x = h + r cos θ and y = k + r sin θ,(Here, 0 ≤ θ ≤ 2π.)Applications of Parametric EquationsThe parametric equations have tremendous applications in a variety of fields. Some of them are as follows:The parametric equations play a key role in kinematics. Here, the paths of the objects through the space can be commonly described as parametric curves. In kinematics, the parametric equations for the coordinates of the given object collectively form a vector-valued function. These curves are further differentiated and integrated termwise.Parametric equations are also used in the field of computer-aided design. For instance, parametric equations allow the users to enjoy the benefit of both the implicit and explicit representations. They help in generating and plotting points on a curve easily and effectively.In integer geometry, various problems can be solved by referring to parametric equations. One of the widely used examples is Euclid's parameterization for the right-angled triangles.ConclusionThe term 'circle' can be defined as a collection of curvy points with a radius equidistant from the centre to the curve of the circle. The parametric equations are one of the most important topics in geometry. Using the parametric equations, we can define the points and coordinates on curves. In short, we can define the coordinates on a circle, hyperbola, parabola, and ellipse. The parametric equations for a standard circle and general circle are different as the standard circle is centred at the origin while the general circle is not centred at the origin. Hope this parametric equations study material helps crack your exams with ease. In this section, we are going to learn, how to find the equation of a line tangent to the curve with parametric equations.Consider a curve with the parametric equations as shown below.x = f(t)y = g(t)where t is the parameter.Equation of a line tangent to the curve :y - y1 = m(x - x1) ----(1)Here,m ---> slope of the tangent line(x1, y1) ----> point of tangencyTo find the slope m, find $^{dy}/_{dx}$ using the formula given below and substitute the value of the parameter t at which the equation of tangent line is required. Substitute the value of the parameter t into the given parametric equations to find the point (x1, y1).Now, substitute the point (x1, y1) and the value m into (1) to get the equation of the tangent line. Problem 1 :Find the equation of the line tangent to the curve with parametric equationsx = 3ty = t2 + 1at t = 2.Solution : To find the value of the slope m, substutite t = 2 into $^{dy}/_{dx}$. Substitute t = 2 into the given parametric equations to find the point of tangency (x1, y1). x = 3tx = 3(2)x = 6 y = t2 + 1y = 22 + 1y = 5 (x1, y1) = (6, 5)Equation of the line tangent to the curve :y - y1 = m(x - x1)Substitute (x1, y1) = (6, 5) and m = ⅔. Problem 2 :Find the equation of the line tangent to the curve with parametric equationsx = 4cosθy = 9sinθat θ = π/4.Solution : To find the value of the slope m, substutite θ = π/4 into $^{dy}/_{dx}$. Substitute θ = π/4 into the given parametric equations to find the point of tangency (x1, y1). (x1, y1) = (6, 5)Equation of the line tangent to the curve :y - y1 = m(x - x1) Problem 3 :Find the equation of the line tangent to the curve with parametric equationsx = t3 - 3ty = t2 - 5tat t = 4.Solution : To find the value of the slope m, substutite t = 4 into $^{dy}/_{dx}$. Substitute t = 4 into the given parametric equations to find the point of tangency (x1, y1). x = t3 - 3tx = 43 - 3(4)x = 64 - 12x = 52 y = t2 - 5ty = 42 - 5(4)y = 16 - 20y = -4 (x1, y1) = (52, -4)Equation of the line tangent to the curve :y - y1 = m(x - x1)Substitute (x1, y1) = (52, -4) and m = $^{1}/_{5}$. Problem 4 :Find the equation of the line tangent to the curve with parametric equationsx = ln ty = t2at t = e.Solution : To find the value of the slope m, substutite t = e into $^{dy}/_{dx}$.m = 2e2Substitute t = 4 into the given parametric equations to find the point of tangency (x1, y1). x = ln tx = ln ex = 1 y = t2y = e2 (x1, y1) = (1, e2)Equation of the line tangent to the curve :y - y1 = m(x - x1)Substitute (x1, y1) = (1, e2) and m = 2e2.y - e2 = 2e2(x - 1)y - e2 = 2e2x - 2e2y = 2e2x - 2e2 + e2y = 2e2x - e2 Problem 5 :Determine the values of t where the tangent lines to the curve with parametric equationsx = t3 - 3ty = t2 - 5tare horizontal and where they are vertical.Solution : If the tangent line to the curve is horizontal, change on vertical axis with respect to t is zero. Since y represents the vertical axis, we have 2t - 5 = 02t = 5t = ⅝Therefore, the tangent line to the curve is horizontal, when t = ⅝If the tangent line to the curve is vertical, change on horizontal axis with respect to t is zero. Since x represents the horizontal axis, we have 3t2 - 3 = 03(t2 - 1) = 0t2 - 1 = 0t2 = 1Taking Square root on both sides,t = ±1Therefore, the tangent line to the curve is vertical, when t = ±1 Kindly mail your feedback to v4formath@gmail.comWe always appreciate your feedback. ©All rights reserved. onlinemath4all.com Find parametric equations for the tangent line at the point $(\cos(-\frac{4 \pi}{6}), \sin(-\frac{4 \pi}{6}), -\frac{4 \pi}{6}))$ on the curve $x = \cos(t), y = \sin(t), z=t$ I understand that in order to find the solution, I need to use partial derivatives. However, the method in my textbook works for simpler problems -- I seem to be making a calculation error when I try to apply the method to this problem. Can anyone suggest how to approach this problem? I found a very similar problem and solution here, but the solution by the person who answered is hard for me to follow. Unfortunately, I get stuck at the line where he subtracts $\frac{\pi}{6}$ from π within the trigonometric functions. Here is the "simple" method that I was originally using. Any sincere help would be appreciated. Thank you. Find parametric equations for the tangent line at the point $(\cos(-\frac{4 \pi}{6}), \sin(-\frac{4 \pi}{6}), -\frac{4 \pi}{6}))$ on the curve $x = \cos(t), y = \sin(t), z=t$ I understand that in order to find the solution, I need to use partial derivatives. However, the method in my textbook works for simpler problems -- I seem to be making a calculation error when I try to apply the method to this problem. Can anyone suggest how to approach this problem? I found a very similar problem and solution here, but the solution by the person who answered is hard for me to follow. Unfortunately, I get stuck at the line where he subtracts $\frac{\pi}{6}$ from π within the trigonometric functions. Here is the "simple" method that I was originally using. Any sincere help would be appreciated. Thank you. Show Mobile Notice Show All Notes Hide All Notes Mobile Notice You appear to be on a device with a "narrow" screen width (i.e. you are probably on a mobile phone). Due to the nature of the mathematics on this site it is best viewed in landscape mode. If your device is not in landscape mode many of the equations will run off the side of your device (you should be able to scroll/swipe to see them) and some of the menu items will be cut off due to the narrow screen width. In this section we want to find the tangent lines to the parametric equations given by, \[x = f\left(t \right)\hspace{0.5in}y = g\left(t \right)\] To do this let's first recall how to find the tangent line to \(y = F\left(x \right)\) at \(x = a\). Here the tangent line is given by, \[y = F\left(a \right) + m\left({x - a} \right),{\mbox{ where }}m = {\left. {\frac{{dy}}{{dx}}} \right|_{x = a}} = F'\left(a \right)\] Now, notice that if we could figure out how to get the derivative \(\frac{{dy}}{{dx}}\) from the parametric equations we could simply reuse this formula since we will be able to use the parametric equations to find the \(x\) and \(y\) coordinates of the point. So, just for a second let's suppose that we were able to eliminate the parameter from the parametric form and write the parametric equations in the form \(y = F\left(x \right)\). Now, plug the parametric equations in for \(x\) and \(y\). Yes, it seems silly to eliminate the parameter, then immediately put it back in, but it's what we need to do in order to get our hands on the derivative. Doing this gives, \[g\left(t \right) = F\left({f\left(t \right)} \right)\] Now, differentiate with respect to \(t\) and notice that we'll need to use the Chain Rule on the right-hand side. \[g'\left(t \right) = F'\left({f\left(t \right)} \right)\,\,f'\left(t \right)\] Let's do another change in notation. We need to be careful with our derivatives here. Derivatives of the lower case function are with respect to \(t\) while derivatives of upper case functions are with respect to \(x\). So, to make sure that we keep this straight let's rewrite things as follows. \[\frac{{dy}}{{dt}} = F'\left(x \right)\frac{{dx}}{{dt}}\] At this point we should remind ourselves just what we are after. We needed a formula for \(\frac{{dy}}{{dx}}\) or \(F'\left(x \right)\) that is in terms of the parametric formulas. Notice however that we can get that from the above equation. Derivative for Parametric Equations \[\frac{{dy}}{{dx}} = \frac{{\displaystyle \,\,\frac{{dy}}{{dt}}\,\,}}{{\displaystyle \,\,\frac{{dx}}{{dt}}\,\,}},{\mbox{ provided }}\frac{{dx}}{{dt}} e 0\] Notice as well that this will be a function of \(t\) and not \(x\). As an aside, notice that we could also get the following formula with a similar derivation if we needed to, \[\frac{{dx}}{{dy}} = \frac{{\displaystyle \,\,\frac{{dx}}{{dt}}\,\,}}{{\displaystyle \,\,\frac{{dy}}{{dt}}\,\,}},{\mbox{ provided }}\frac{{dy}}{{dt}} e 0\] Why would we want to do this? Well, recall that in the arc length section of the Applications of Integral section we actually needed this derivative on occasion. So, let's find a tangent line. Example 1 Find the tangent line(s) to the parametric curve given by \[x = {t^5} - 4{t^3}\hspace{0.5in}y = {t^2}\] at \(\left({0,4} \right)\). Show Solution Note that there is apparently the potential for more than one tangent line here! We will look into this more after we're done with the example. The first thing that we should do is find the derivative so we can get the slope of the tangent line. \[\frac{{dy}}{{dx}} = \frac{{\displaystyle \,\,\frac{{dy}}{{dt}}\,\,}}{{\displaystyle \,\,\frac{{dx}}{{dt}}\,\,}} = \frac{{2t}}{{5{t^4} - 12{t^2}}} = \frac{2}{{5{t^3} - 12t}}\] At this point we've got a small problem. The derivative is in terms of \(t\) and all we've got is an x-y coordinate pair. The next step then is to determine the value(s) of \(t\) which will give this point. We find these by plugging the \(x\) and \(y\) values into the parametric equations and solving for \(t\). \[\begin{align*}0 &= {t^5} - 4{t^3} = {t^3}\left({{t^2} - 4} \right) &\Rightarrow \hspace{0.25in} & t = 0, \pm 2\\ 4 &= {t^2} & \Rightarrow \hspace{0.25in} & t = \pm 2\end{align*}\] Any value of \(t\) which appears in both lists will give the point. So, since there are two values of \(t\) that give the point we will in fact get two tangent lines. That's definitely not something that happened back in Calculus I and we're going to need to look into this a little more. However, before we do that let's actually get the tangent lines. \(t = - 2:\)Since we already know the \(x\) and \(y\)-coordinates of the point all that we need to do is find the slope of the tangent line. \[m = {\left. {\frac{{dy}}{{dx}}} \right|_{\,t\, = \, - 2}} = - \frac{1}{8}\] The tangent line (at \(t = - 2\)) is then, \[y = 4 - \frac{1}{8}x\] \(t = 2:\)Again, all we need is the slope. \[m = {\left. {\frac{{dy}}{{dx}}} \right|_{\,t\, = \,2}} = \frac{1}{8}\] The tangent line (at \(t = 2\)) is then, \[y = 4 + \frac{1}{8}x\] Before we leave this example let's take a look at just how we could possibly get two tangent lines at a point. This was definitely not possible back in Calculus I where we first ran across tangent lines. A quick graph of the parametric curve will explain what is going on here. So, the parametric curve crosses itself! That explains how there can be more than one tangent line. There is one tangent line for each instance that the curve goes through the point. The next topic that we need to discuss in this section is that of horizontal and vertical tangents. We can easily identify where these will occur (or at least the \(t\)'s that will give them) by looking at the derivative formula. \[\frac{{dy}}{{dx}} = \frac{{\displaystyle \,\,\frac{{dy}}{{dt}}\,\,}}{{\displaystyle \,\,\frac{{dx}}{{dt}}\,\,}}\] Horizontal tangents will occur where the derivative is zero and that means that we'll get horizontal tangent at values of \(t\) for which we have, Horizontal Tangent for Parametric Equations \[\frac{{dy}}{{dt}} = 0,{\mbox{ provided }}\frac{{dx}}{{dt}} e 0\] Vertical tangents will occur where the derivative is not defined and so we'll get vertical tangents at values of \(t\) for which we have, Vertical Tangent for Parametric Equations \[\frac{{dx}}{{dt}} = 0,{\mbox{ provided }}\frac{{dy}}{{dt}} e 0\] Let's take a quick look at an example of this. Example 2 Determine the \(x\)-\(y\) coordinates of the points where the following parametric equations will have horizontal or vertical tangents. \[x = {t^3} - 3t\hspace{0.5in}y = 3{t^2} - 9\] Show Solution We'll first need the derivatives of the parametric equations. \[\frac{{dx}}{{dt}} = 3{t^2} - 3 = 3\left({{t^2} - 1} \right)\hspace{0.5in}\frac{{dy}}{{dt}} = 6t\] Horizontal TangentsWe'll have horizontal tangents where, \[6t = 0\hspace{0.5in} \Rightarrow \hspace{0.5in}t = 0\] Now, this is the value of \(t\) which gives the horizontal tangents and we were asked to find the x-y coordinates of the point. To get these we just need to plug \(t\) into the parametric equations. Therefore, the only horizontal tangent will occur at the point \(\left({0, - 9} \right)\). Vertical TangentsIn this case we need to solve, \[3\left({{t^2} - 1} \right) = 0\hspace{0.5in} \Rightarrow \hspace{0.5in}t = \pm 1\] The two vertical tangents will occur at the points \(\left({2, - 6} \right)\) and \(\left({ - 2, - 6} \right)\). For the sake of completeness and at least partial verification here is the sketch of the parametric curve. The final topic that we need to discuss in this section really isn't related to tangent lines but does fit in nicely with the derivative of the derivative that we needed to get the slope of the tangent line. Before moving into the new topic let's first remind ourselves of the formula for the first derivative and in the process rewrite it slightly. \[\frac{{dy}}{{dx}} = \frac{d}{{dx}}\left(y \right) = \frac{{\displaystyle \frac{d}{{dt}}\left(y \right)}}{{\displaystyle \frac{{dx}}{{dt}}}}\] Written in this way we can see that the formula actually tells us how to differentiate a function \(y\) (as a function of \(t\)) with respect to \(x\) (when \(x\) is also a function of \(t\)) when we are using parametric equations. Now let's move onto the final topic of this section. We would also like to know how to get the second derivative of \(y\) with respect to \(x\). \[\frac{{{d^2}y}}{{d{x^2}}}\] Getting a formula for this is fairly simple if we remember the rewritten formula for the first derivative above. Second Derivative for Parametric Equations \[\frac{{{d^2}y}}{{d{x^2}}} = \frac{d}{{dx}}\left({\frac{{dy}}{{dx}}} \right) = \frac{{\displaystyle \frac{d}{{dt}}\left({\frac{{dy}}{{dx}}} \right)}}{{\displaystyle \frac{{dx}}{{dt}}}}\] It is important to note that, \[\frac{{{d^2}y}}{{d{x^2}}} e \frac{{\displaystyle \,\,\frac{{{d^2}y}}{{d{t^2}}}\,\,}}{{\displaystyle \,\,\frac{{{d^2}x}}{{d{t^2}}}\,\,}}\] Let's work a quick example. Example 3 Find the second derivative for the following set of parametric equations. \[x = {t^5} - 4{t^3}\hspace{0.5in}y = {t^2}\] Show Solution This is the set of parametric equations that we used in the first example and so we already have the following computations completed. \[\frac{{dy}}{{dt}} = 2t\hspace{0.5in}\hspace{0.25in}\frac{{dx}}{{dt}} = 5{t^4} - 12{t^2}\hspace{0.5in}\frac{{dy}}{{dx}} = \frac{2}{{5{t^3} - 12t}}\] We will first need the following, \[\frac{d}{{dt}}\left({\frac{2}{{5{t^3} - 12t}}} \right) = \frac{{ - 2\left({15{t^2} - 12} \right)}}{{{{\left({5{t^3} - 12t} \right)}^2}}} = \frac{24 - 30{t^2}}{{{{\left({5{t^3} - 12t} \right)}^2}}}\] The second derivative is then, \[\begin{align*}\frac{{{d^2}y}}{{d{x^2}}} & = \frac{{\frac{d}{{dt}}\left({\frac{{dy}}{{dx}}} \right)}}{{\frac{{dx}}{{dt}}}}\\ & = \frac{{\frac{24 - 30{t^2}}{{{{\left({5{t^3} - 12t} \right)}^2}}}}}{{5{t^4} - 12{t^2}}}\\ & = \frac{24 - 30{t^2}}{{{\left({5{t^4} - 12{t^2}} \right){\left({5{t^3} - 12t} \right)}^2}}}\\ & = \frac{24 - 30{t^2}}{{{\left({5{t^4} - 12{t^2}} \right){\left({5{t^3} - 12t} \right)}^2}}}\\ & = \frac{24 - 30{t^2}}{{t{{\left({5{t^3} - 12t} \right)}^3}}}\end{align*}\] So, why would we want the second derivative? Well, recall from your Calculus I class that with the second derivative we can determine where a curve is concave up and concave down. We could do the same thing with parametric equations if we wanted to. Example 4 Determine the values of \(t\) for which the parametric curve given by the following set of parametric equations is concave up and concave down. \[x = 1 - {t^2}\hspace{0.5in}y = {t^7} + {t^5}\] Show Solution To compute the second derivative we'll first need the following. \[\frac{{dy}}{{dt}} = 7{t^6} + 5{t^4}\hspace{0.5in}\frac{{dy}}{{dx}} = - 2t\hspace{0.5in}\frac{{dy}}{{dx}} = \frac{{7{t^6} + 5{t^4}}}{{ - 2t}} = - \frac{1}{2}\left({7{t^5} + 5{t^3}} \right)\] Note that we can also use the first derivative above to get some information about the increasing/decreasing nature of the curve as well. In this case it looks like the parametric curve will be increasing if \(t < 0\) and decreasing if \(t > 0\). Now let's move on to the second derivative. \[\frac{{{d^2}y}}{{d{x^2}}} = \frac{{ - \frac{1}{2}\left({35{t^4} + 15{t^2}} \right)}}{{ - 2t}} = \frac{1}{4}\left({35{t^3} + 15t} \right)\] It's clear, hopefully, that the second derivative will only be zero at \(t = 0\). Using this we can see that the second derivative will be negative if \(t < 0\) and positive if \(t > 0\). So the parametric curve will be concave down for \(t < 0\) and concave up for \(t > 0\). Here is a sketch of the curve for completeness sake. Show Mobile Notice Show All Notes Hide All Notes Mobile Notice You appear to be on a device with a "narrow" screen width (i.e. you are probably on a mobile phone). Due to the nature of the mathematics on this site it is best viewed in landscape mode. If your device is not in landscape mode many of the equations will run off the side of your device (you should be able to scroll/swipe to see them) and some of the menu items will be cut off due to the narrow screen width. In this section we want to find the tangent lines to the parametric equations given by, \[x = f\left(t \right)\hspace{0.5in}y = g\left(t \right)\] To do this let's first recall how to find the tangent line to \(y = F\left(x \right)\) at \(x = a\). Here the tangent line is given by, \[y = F\left(a \right) + m\left({x - a} \right),{\mbox{ where }}m = {\left. {\frac{{dy}}{{dx}}} \right|_{x = a}} = F'\left(a \right)\] Now, notice that if we could figure out how to get the derivative \(\frac{{dy}}{{dx}}\) from the parametric equations we could simply reuse this formula since we will be able to use the parametric equations to find the \(x\) and \(y\) coordinates of the point. So, just for a second let's suppose that we were able to eliminate the parameter from the parametric form and write the parametric equations in the form \(y = F\left(x \right)\). Now, plug the parametric equations in for \(x\) and \(y\). Yes, it seems silly to eliminate the parameter, then immediately put it back in, but it's what we need to do in order to get our hands on the derivative. Doing this gives, \[g\left(t \right) = F\left({f\left(t \right)} \right)\] Now, differentiate with respect to \(t\) and notice that we'll need to use the Chain Rule on the right-hand side. \[g'\left(t \right) = F'\left({f\left(t \right)} \right)\,\,f'\left(t \right)\] Let's do another change in notation. We need to be careful with our derivatives here. Derivatives of the lower case function are with respect to \(t\) while derivatives of upper case functions are with respect to \(x\). So, to make sure that we keep this straight let's rewrite things as follows. \[\frac{{dy}}{{dt}} = F'\left(x \right)\frac{{dx}}{{dt}}\] At this point we should remind ourselves just what we are after. We needed a formula for \(\frac{{dy}}{{dx}}\) or \(F'\left(x \right)\) that is in terms of the parametric formulas. Notice however that we can get that from the above equation. Derivative for Parametric Equations \[\frac{{dy}}{{dx}} = \frac{{\displaystyle \,\,\frac{{dy}}{{dt}}\,\,}}{{\displaystyle \,\,\frac{{dx}}{{dt}}\,\,}},{\mbox{ provided }}\frac{{dx}}{{dt}} e 0\] Notice as well that this will be a function of \(t\) and not \(x\). As an aside, notice that we could also get the following formula with a similar derivation if we needed to, \[\frac{{dx}}{{dy}} = \frac{{\displaystyle \,\,\frac{{dx}}{{dt}}\,\,}}{{\displaystyle \,\,\frac{{dy}}{{dt}}\,\,}},{\mbox{ provided }}\frac{{dy}}{{dt}} e 0\] Why would we want to do this? Well, recall that in the arc length section of the Applications of Integral section we actually needed this derivative on occasion. So, let's find a tangent line. Example 1 Find the tangent line(s) to the parametric curve given by \[x = {t^5} - 4{t^3}\hspace{0.5in}y = {t^2}\] at \(\left({0,4} \right)\). Show Solution Note that there is apparently the potential for more than one tangent line here! We will look into this more after we're done with the example. The first thing that we should do is find the derivative so we can get the slope of the tangent line. \[\frac{{dy}}{{dx}} = \frac{{\displaystyle \,\,\frac{{dy}}{{dt}}\,\,}}{{\displaystyle \,\,\frac{{dx}}{{dt}}\,\,}} = \frac{{2t}}{{5{t^4} - 12{t^2}}} = \frac{2}{{5{t^3} - 12t}}\] At this point we've got a small problem. The derivative is in terms of \(t\) and all we've got is an x-y coordinate pair. The next step then is to determine the value(s) of \(t\) which will give this point. We find these by plugging the \(x\) and \(y\) values into the parametric equations and solving for \(t\). \[\begin{align*}0 &= {t^5} - 4{t^3} = {t^3}\left({{t^2} - 4} \right) &\Rightarrow \hspace{0.25in} & t = 0, \pm 2\\ 4 &= {t^2} & \Rightarrow \hspace{0.25in} & t = \pm 2\end{align*}\] Any value of \(t\) which appears in both lists will give the point. So, since there are two values of \(t\) that give the point we will in fact get two tangent lines. That's definitely not something that happened back in Calculus I and we're going to need to look into this a little more. However, before we do that let's actually get the tangent lines. \(t = - 2:\)Since we already know the \(x\) and \(y\)-coordinates of the point all that we need to do is find the slope of the tangent line. \[m = {\left. {\frac{{dy}}{{dx}}} \right|_{\,t\, = \, - 2}} = - \frac{1}{8}\] The tangent line (at \(t = - 2\)) is then, \[y = 4 - \frac{1}{8}x\] \(t = 2:\)Again, all we need is the slope. \[m = {\left. {\frac{{dy}}{{dx}}} \right|_{\,t\, = \,2}} = \frac{1}{8}\] The tangent line (at \(t = 2\)) is then, \[y = 4 + \frac{1}{8}x\] Before we leave this example let's take a look at just how we could possibly get two tangent lines at a point. This was definitely not possible back in Calculus I where we first ran across tangent lines. A quick graph of the parametric curve will explain what is going on here. So, the parametric curve crosses itself! That explains how there can be more than one tangent line. There is one tangent line for each instance that the curve goes through the point. The next topic that we need to discuss in this section is that of horizontal and vertical tangents. We can easily identify where these will occur (or at least the \(t\)'s that will give them) by looking at the derivative formula. \[\frac{{dy}}{{dx}} = \frac{{\displaystyle \,\,\frac{{dy}}{{dt}}\,\,}}{{\displaystyle \,\,\frac{{dx}}{{dt}}\,\,}}\] Horizontal tangents will occur where the derivative is zero and that means that we'll get horizontal tangent at values of \(t\) for which we have, Horizontal Tangent for Parametric Equations \[\frac{{dy}}{{dt}} = 0,{\mbox{ provided }}\frac{{dx}}{{dt}} e 0\] Vertical tangents will occur where the derivative is not defined and so we'll get vertical tangents at values of \(t\) for which we have, Vertical Tangent for Parametric Equations \[\frac{{dx}}{{dt}} = 0,{\mbox{ provided }}\frac{{dy}}{{dt}} e 0\] Let's take a quick look at an example of this. Example 2 Determine the \(x\)-\(y\) coordinates of the points where the following parametric equations will have horizontal or vertical tangents. \[x = {t^3} - 3t\hspace{0.5in}y = 3{t^2} - 9\] Show Solution We'll first need the derivatives of the parametric equations. \[\frac{{dx}}{{dt}} = 3{t^2} - 3 = 3\left({{t^2} - 1} \right)\hspace{0.5in}\frac{{dy}}{{dt}} = 6t\] Horizontal TangentsWe'll have horizontal tangents where, \[6t = 0\hspace{0.5in} \Rightarrow \hspace{0.5in}t = 0\] Now, this is the value of \(t\) which gives the horizontal tangents and we were asked to find the x-y coordinates of the point. To get these we just need to plug \(t\) into the parametric equations. Therefore, the only horizontal tangent will occur at the point \(\left({0, - 9} \right)\). Vertical TangentsIn this case we need to solve, \[3\left({{t^2} - 1} \right) = 0\hspace{0.5in} \Rightarrow \hspace{0.5in}t = \pm 1\] The two vertical tangents will occur at the points \(\left({2, - 6} \right)\) and \(\left({ - 2, - 6} \right)\). For the sake of completeness and at least partial verification here is the sketch of the parametric curve. The final topic that we need to discuss in this section really isn't related to tangent lines but does fit in nicely with the derivative of the derivative that we needed to get the slope of the tangent line. Before moving into the new topic let's first remind ourselves of the formula for the first derivative and in the process rewrite it slightly. \[\frac{{dy}}{{dx}} = \frac{d}{{dx}}\left(y \right) = \frac{{\displaystyle \frac{d}{{dt}}\left(y \right)}}{{\displaystyle \frac{{dx}}{{dt}}}}\] Written in this way we can see that the formula actually tells us how to differentiate a function \(y\) (as a function of \(t\)) with respect to \(x\) (when \(x\) is also a function of \(t\)) when we are using parametric equations. Now let's move onto the final topic of this section. We would also like to know how to get the second derivative of \(y\) with respect to \(x\). \[\frac{{{d^2}y}}{{d{x^2}}}\] Getting a formula for this is fairly simple if we remember the rewritten formula for the first derivative above. Second Derivative for Parametric Equations \[\frac{{{d^2}y}}{{d{x^2}}} = \frac{d}{{dx}}\left({\frac{{dy}}{{dx}}} \right) = \frac{{\displaystyle \frac{d}{{dt}}\left({\frac{{dy}}{{dx}}} \right)}}{{\displaystyle \frac{{dx}}{{dt}}}}\] It is important to note that, \[\frac{{{d^2}y}}{{d{x^2}}} e \frac{{\displaystyle \,\,\frac{{{d^2}y}}{{d{t^2}}}\,\,}}{{\displaystyle \,\,\frac{{{d^2}x}}{{d{t^2}}}\,\,}}\] Let's work a quick example. Example 3 Find the second derivative for the following set of parametric equations. \[x = {t^5} - 4{t^3}\hspace{0.5in}y = {t^2}\] Show Solution This is the set of parametric equations that we used in the first example and so we already have the following computations completed. \[\frac{{dy}}{{dt}} = 2t\hspace{0.5in}\hspace{0.25in}\frac{{dx}}{{dt}} = 5{t^4} - 12{t^2}\hspace{0.5in}\frac{{dy}}{{dx}} = \frac{2}{{5{t^3} - 12t}}\] We will first need the following, \[\frac{d}{{dt}}\left({\frac{2}{{5{t^3} - 12t}}} \right) = \frac{{ - 2\left({15{t^2} - 12} \right)}}{{{{\left({5{t^3} - 12t} \right)}^2}}} = \frac{24 - 30{t^2}}{{{{\left({5{t^3} - 12t} \right)}^2}}}\] The second derivative is then, \[\begin{align*}\frac{{{d^2}y}}{{d{x^2}}} & = \frac{{\frac{d}{{dt}}\left({\frac{{dy}}{{dx}}} \right)}}{{\frac{{dx}}{{dt}}}}\\ & = \frac{{\frac{24 - 30{t^2}}{{{{\left({5{t^3} - 12t} \right)}^2}}}}}{{5{t^4} - 12{t^2}}}\\ & = \frac{24 - 30{t^2}}{{{\left({5{t^4} - 12{t^2}} \right){\left({5{t^3} - 12t} \right)}^2}}}\\ & = \frac{24 - 30{t^2}}{{{\left({5{t^4} - 12{t^2}} \right){\left({5{t^3} - 12t} \right)}^2}}}\\ & = \frac{24 - 30{t^2}}{{t{{\left({5{t^3} - 12t} \right)}^3}}}\end{align*}\] So, why would we want the second derivative? Well, recall from your Calculus I class that with the second derivative we can determine where a curve is concave up and concave down. We could do the same thing with parametric equations if we wanted to. Example 4 Determine the values of \(t\) for which the parametric cur